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RENORMALIZATION CONSTANTS

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# Scaling behavior of improvement and renormalization constants \*

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This talk summarizes results for all the scale independent renormalization constants for bilinear currents ( $Z_A$ ,  $Z_V$ , and  $Z_S/Z_P$ ), the improvement constants ( $c_A$ ,  $c_V$ , and  $c_T$ ), the quark mass dependence of  $Z_O$ , and the coefficients of the equation of motion operators for  $O(a)$  improved lattice QCD. Using data at  $\beta = 6.0, 6.2$  and  $6.4$  we study the scaling behavior of these quantities and quantify residual discretization errors.

The use of axial and vector Ward identities has proven to be a very efficient and reliable way of extracting the improvement and renormalization constants for the  $O(a)$  improved fermion action. The methodology, references to previous calculations, and the notation we use are given in [1]. The features of the calculation summarized here are: new data at  $\beta = 6.4$ ; a reanalysis of  $c_A$  including  $O(m^2a^2)$  corrections and extraction of  $c_A$  from non-zero momentum correlators; improved chiral extrapolations of  $Z_A^0$ ,  $c_T$ ,  $\tilde{b}_P - \tilde{b}_A$ ; scaling behavior of all the constants including comparison against results by ALPHA collaboration and 1-loop perturbation theory (see [1] and these estimates are also summarized in Table 1). Detailed results will be presented in [2].

The new data at  $\beta = 6.4$  is obtained on 60 lattices of size  $24^3 \times 64$ . The region of chiral rotation is selected to be between time slices 9 – 57 with the source at  $t = 1$ . This single insertion region allows us to gather data using both forward and backward propagation of states, and represents an improvement over calculations reported in [1]. Overall, we find that there is a tremendous improvement in the quality of the signal with  $\beta$ . The data at  $\beta = 6.4$  has allowed us to resolve certain trends we saw earlier.

The first such feature is the need for including an  $O(m^2a^2)$  term in the extrapolation of  $c_A$  to the chiral limit. Data at  $\beta = 6.4$  is shown in Fig. 1. In Table 1 we give results of both the

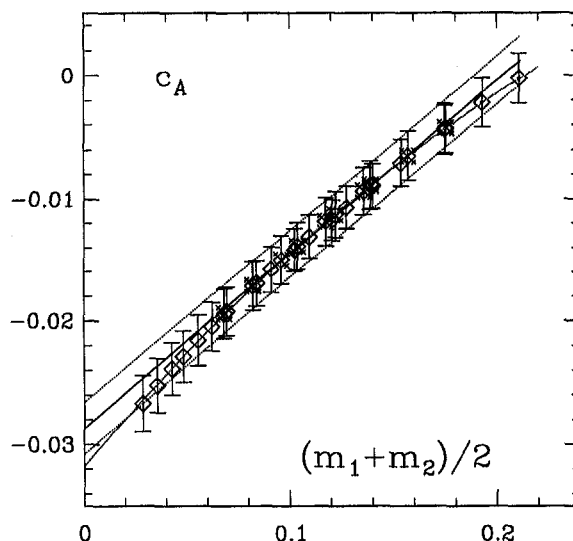


Figure 1. Comparison of linear and quadratic extrapolation of  $c_A$  to the chiral limit

linear and quadratic fit (marked by an asterisk), and take the quadratic fit as the preferred final value. Our results show a weak dependence of  $c_A$  on  $\beta$  in the range  $6.0 - 6.4$ , unlike that found by the ALPHA collaboration, but consistent with the recent results by Collins *et al.* [3].

The second point concerns the chiral extrapolation for  $Z_A^0$ ,  $c_T$ ,  $\tilde{b}_P - \tilde{b}_A$ . Our estimates presented in [1] were based on constant fits as these quantities are not expected to have  $O(ma)$  corrections if the theory is fully improved to  $O(a)$ . We now choose to advocate using results of linear extrapolation as our data show a dependence on  $m$  which can arise due to terms of the form  $O(a\Lambda_{QCD}ma)$  as we have used mass-dependent

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Table 1

The first error in LANL estimates is statistical, and the second, where present, corresponds to the difference between using 2-point and 3-point discretization of the derivative in extraction of  $c_A$ . Asterisk marks our preferred values which include  $O(ma)$  corrections in the chiral extrapolations.

	$\beta = 6.0$			$\beta = 6.2$			$\beta = 6.4$		
	LANL	ALPHA	P. Th.	LANL	ALPHA	P. Th.	LANL	ALPHA	P. Th.
$c_{SW}$	1.769	1.769	1.521	1.614	1.614	1.481	1.526	1.526	1.449
$Z_V^0$	+0.770(1)	+0.7809(6)	+0.810	+0.7874(4)	+0.7922(4)(9)	+0.821	+0.802(1)	+0.8032(6)(12)	+0.830
$Z_A^0$	+0.807(2)(8)	+0.7906(94)	+0.829	+0.818(2)(5)	+0.807(8)(2)	+0.839	+0.827(1)(4)	+0.827(8)(1)	+0.847
$Z_A^{0*}$	+0.802(2)(8)			+0.815(2)(5)			+0.822(1)(4)		
$Z_P^0/Z_S^0$	+0.842(5)(1)	N.A.	+0.956	+0.884(3)(1)	N.A.	+0.959	+0.901(2)(5)	N.A.	+0.962
$c_A$	-0.037(4)(8)	-0.083(5)	-0.013	-0.032(3)(6)	-0.038(4)	-0.012	-0.029(2)(4)	-0.025(2)	-0.011
$c_A^*$	-0.038(4)			-0.033(3)			-0.032(3)		
$c_V$	-0.107(17)(4)	-0.32(7)	-0.028	-0.09(2)(1)	-0.21(7)	-0.026	-0.08(1)(2)	-0.13(5)	-0.024
$c_T$	+0.063(7)(29)	N.A.	+0.020	+0.051(7)(17)	N.A.	+0.019	+0.041(3)(23)	N.A.	+0.018
$c_T^*$	+0.076(10)			+0.059(8)			+0.051(4)		
$\tilde{b}_V$	+1.43(1)(4)	N.A.	+1.106	+1.30(1)(1)	N.A.	+1.099	+1.24(1)(1)	N.A.	+1.093
$b_V$	+1.52(1)	+1.54(2)	+1.274	+1.42(1)	+1.41(2)	+1.255	+1.39(1)	+1.36(3)	+1.239
$\tilde{b}_A - \tilde{b}_V$	-0.26(3)(4)	N.A.	-0.002	-0.11(3)(4)	N.A.	-0.002	-0.09(1)(1)	N.A.	-0.002
$b_A - b_V$	-0.24(3)(4)	N.A.	-0.002	-0.11(3)(4)	N.A.	-0.002	-0.08(1)(1)	N.A.	-0.002
$\tilde{b}_P - \tilde{b}_S$	-0.06(4)(3)	N.A.	-0.066	-0.09(2)(1)	N.A.	-0.062	-0.090(10)(1)	N.A.	-0.059
$\tilde{b}_P - \tilde{b}_A$	-0.07(4)(5)	N.A.	+0.002	-0.09(3)(3)	N.A.	+0.001	-0.12(2)(5)	N.A.	+0.001
$\tilde{b}_P - \tilde{b}_A^*$	-0.08(30)			+0.03(9)			-0.02(4)		
$\tilde{b}_A$	+1.17(4)(8)	N.A.	+1.104	+1.19(3)(5)	N.A.	+1.097	+1.16(2)(3)	N.A.	+1.092
$b_A$	+1.28(3)(4)	N.A.	+1.271	+1.32(3)(4)	N.A.	+1.252	+1.31(2)(1)	N.A.	+1.237
$\tilde{b}_P$	+1.10(5)(13)	N.A.	+1.105	+1.11(4)(7)	N.A.	+1.099	+1.04(3)(7)	N.A.	+1.093
$\tilde{b}_S$	+1.16(6)(11)	N.A.	+1.172	+1.19(4)(6)	N.A.	+1.161	+1.13(3)(8)	N.A.	+1.151

value of  $c_A$  in intermediate stages of these calculations. Even though a fit linear in  $ma$  removes only part of the  $O(a^2)$  corrections, we choose it as our preferred value (marked with an asterisk) as it is less sensitive to the  $m$  values used in the fit. To show the size of this effect, we give both estimates in Table 1. One exception is  $\tilde{b}_P - \tilde{b}_A$  at  $\beta = 6.0$  for which the constant fit is our preferred estimate as the data do not show a linear term.

The third new feature is the demonstration that consistent estimates for  $c_A$  are obtained from correlators with zero and non-zero momentum once additional  $O(p^2 a^2)$  errors are accounted for. Data at  $\beta = 6.4$  is shown in Fig. 2 where we plot  $c_A$  versus  $(12pa/\pi)^2$ . We find that a linear extrapolation to  $p = 0$  yields results consistent with those obtained using zero momentum correlators.

Our estimates for  $Z_A^0$ ,  $Z_V^0$ ,  $c_A$ ,  $c_V$ , and  $b_V$

can be compared against those obtained by the ALPHA collaboration who used the Schrodinger functional method. Estimates for  $b_V$  are already consistent within respective errors at all three  $\beta$  values. For the other four quantities we expect the difference ( $LANL_{preferred} - ALPHA$ ) to vanish as  $O(a^2)$  for  $Z_A^0$  and  $Z_V^0$ , and as  $O(a)$  for  $c_A$  and  $c_V$ . Fits assuming this leading behavior and requiring that the difference vanish at  $a = 0$ , give

$$\Delta Z_V^0 = +(33a)^2 - (475a)^3 \quad (1)$$

$$\Delta Z_A^0 = -(246a)^2 + (629a)^3 \quad (2)$$

$$\Delta c_A = -(181a) + (763a)^2 \quad (3)$$

$$\Delta c_V = -(367a) + (669a)^2 \quad (4)$$

where  $a$  is in units of  $(\text{MeV}^{-1})$ . Prior to interpretation, a number of comments are in order

- The differences in  $Z_V^0$  are significant, how-

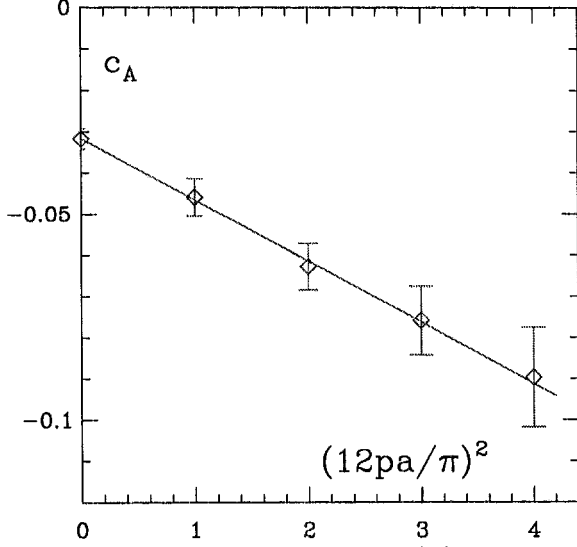


Figure 2. Evidence for additional  $O(p^2 a^2)$  errors in  $c_A$  extracted from  $p \neq 0$  correlators.

ever the fit shows reasonable estimates for the higher order corrections.

- The differences in  $Z_A^0$  are at most one combined  $\sigma$ .
- The coefficients in the fit for  $\Delta c_A$  are dominated by the significant difference at  $\beta = 6.0$ . It is, therefore, important to resolve whether  $c_A$  develops the large  $\beta$  dependence seen by the ALPHA Collaboration below  $\beta = 6.4$ .
- The errors in the ALPHA collaboration estimates of  $c_V$  are a significant fraction of the difference.

These fits, based on data at three  $\beta$  values with  $1/a$  between 2.1 and 3.86 GeV, should be considered indicative and qualitative and certainly not sufficient to draw precise conclusions. To stress this fact we do not give any error estimates for the fit parameters.

Using our three data points we can also fit the difference between the non-perturbative and tadpole improved 1-loop estimates as a function of the leading residual discretization error in  $a$  and perturbative corrections,  $O(\alpha_s)$ , giving

$$\Delta Z_A^0 = -(158a)^2 - (1.4\alpha_s)^2 \quad (5)$$

Table 2

Results for off-shell mixing coefficients.

$\beta$	6.0(f)	6.0(b)	6.2	6.4
$c'_V + c'_P$	2.82(15)	+2.68(19)	2.62(8)	2.44(4)
$c'_A + c'_P$	2.43(24)	+2.12(31)	2.43(14)	2.27(6)
$2c'_P$	0.88(97)	-0.65(57)	1.82(24)	1.85(8)
$c'_S + c'_P$	2.44(13)	+2.40(13)	2.40(7)	2.27(4)
$c'_T + c'_P$	2.40(18)	+2.27(20)	2.42(9)	2.28(5)
$c'_V$	2.38(50)	+3.00(37)	1.72(16)	1.52(4)
$c'_A$	1.99(56)	+2.45(46)	1.53(20)	1.35(6)
$c'_P$	0.44(49)	-0.33(29)	0.91(12)	0.93(4)
$c'_S$	2.00(48)	+2.72(33)	1.49(14)	1.35(4)
$c'_T$	1.96(49)	+2.60(38)	1.51(15)	1.36(4)

$$\Delta Z_V^0 = (197a)^2 - (1.4\alpha_s)^2 \quad (6)$$

$$\Delta Z_P^0/Z_S^0 = -(502a)^2 - (1.8\alpha_s)^2 \quad (7)$$

$$\Delta c_A = -(13a) - (1.3\alpha_s)^2 \quad (8)$$

$$\Delta c_V = -(51a) - (1.7\alpha_s)^2 \quad (9)$$

$$\Delta c_T = (94a) + (0.8\alpha_s)^2 \quad (10)$$

$$\Delta \tilde{b}_V = (1010a) - (2.9\alpha_s)^2 \quad (11)$$

$$\Delta b_V = (429a) + (1.5\alpha_s)^2 \quad (12)$$

where  $a$  is expressed in  $\text{MeV}^{-1}$ ,  $u_0$  is  $\langle \text{plquette} \rangle^{1/4}$ , and  $\alpha_s = g^2/(4\pi u_0^4)$  is the tadpole improved coupling with values 0.1340, 0.1255 and 0.1183 at the three  $\beta$ . The errors in the other  $\tilde{b}$  are too large to allow any meaningful fits.

The discretization error for  $\Delta \tilde{b}_V$  is unexpectedly large; those for  $\Delta b_V$  and  $\Delta Z_P^0/Z_S^0$  about  $2\Lambda_{QCD}a$ ; and reasonable for the rest including the fact that the perturbative corrections are  $\lesssim 2\alpha_s$ . Precise data at other  $\beta$  are required to clarify the scaling behavior of these constants.

Finally, in Table 2 we present results for the coefficients of the equation of motion operators. Estimates at  $\beta = 6.0$  are poor, but become reasonably precise at  $\beta = 6.2$  and 6.4. We find that except for  $c'_P$ , the corrections to the tree level value  $c'_O = 1$  are large.

## REFERENCES

1. T. Bhattacharya *et al.*, Phys. Rev. D63 (2001) 074505.
2. T. Bhattacharya *et al.*, in preparation.

3. S. Collins *et al.*, these proceedings.